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Development of a new moisture transfer (*Bi–Re*) correlation for food drying applications

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Abstract

In this paper, newly developed Biot–Reynolds (*Bi–Re*) correlation to determine the moisture transfer parameters is presented. Development of the new correlation is based on the experimental data taken from various sources in the literature. Moisture diffusivity and moisture transfer coefficient are calculated using the previously developed model. The moisture distribution profiles are then obtained for regular objects such as slab, cylinder and sphere. The results obtained from the present study are compared with the experimental data and a correlation available in the literature. It is found that they are in good agreement. Hence, it is believed that the developed correlation is of great significance for design and practicing engineers working in the drying industries. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

Drying is considered as the oldest food preservation technique, and a common unit operation in many chemical and process industries. The removal of moisture prevents the growth and reproduction of microorganisms causing decay and minimizes many of the moisture mediated deteriorative reactions. Drying is also one of the most widely used unit operation in the industry. No other unit operation exists which is capable of handling such a large variety of products ranging from paper to food. Since drying is an energy intensive operation, and due to the sharp increase in energy cost over the last few years, it has become the prime concern of the researchers to find the means of attaining optimum process conditions for good quality products, which leads to energy savings. Towards this goal, accurate determination of moisture transfer parameters for the drying operation is essential. Moreover, decreasing the energy consumption in the drying process will decrease the environment impact in terms of pollutants and hence protect the environment.

In the literature, numerous experimental and theoretical studies have been carried out on the determination of drying profiles for various food products by many researchers [1–4]. Although there is a large amount of studies available in the literature to determine and calculate the moisture transfer parameters such as moisture diffusivities and moisture transfer coefficients for the products subjected to drying, limited studies have been carried out to determine these parameters using the drying process parameters in terms of lag factor and drying coefficient as first introduced by [5,6]. Recently, El-Naas et al. [7] developed a correlation for estimating the mass transfer coefficients of the products in a spouted bed dryer based on the experimental results. To date, to the best of the authors' knowledge, there is no any study carried out on the development of Biot–Reynolds (*Bi–Re*) correlation for the products, which are subjected to drying. The objective of this study is to present a newly developed *Bi–Re* correlation, validate it with experimental data and discuss its safe use in practical drying applications.

2. Analysis

In this section, we will discuss the transient moisture diffusion involved during the drying process. Consider

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Nomenclature

a, b	constants
A, B	constants
Bi	Biot number for mass transfer
d	diameter
D	moisture diffusivity (m^2/s)
Fo	Fourier number for mass transfer
G	lag factor
$J_0(\mu_1)$	zeroth-order Bessel function of the first kind
$J_1(\mu_1)$	first-order Bessel function of the first kind
K	mass transfer coefficient (m/s)
L	half thickness (m)
M	moisture content (kg/kg)
r	radial correlation
R	radius (m)
S	drying coefficient (1/s)
Sh	Sherwood number
t	time (t)
y	coordinate
z	rectangular coordinate
Greek symbols	
ϕ	moisture content difference (kg/kg)
Φ	dimensionless moisture content
μ	root of the transcendental characteristic equation
Γ	dimensionless distance
Subscripts	
e	equilibrium
i	Initial
m	constant
1	refers to 1st characteristic value

an infinite slab object (ISO), or an infinite cylindrical object (ICO) or a spherical object (SO) subjected to drying in a medium of air for Bi between 0.1 and 100. The transient moisture diffusivity equation in cartesian, cylindrical and spherical coordinates can be written in the following form:

$$\left(\frac{1}{y^m}\right)\left(\frac{\partial}{\partial y}\right)\left[y^m\left(\frac{\partial \phi}{\partial y}\right)\right]=\left(\frac{1}{D}\right)\left(\frac{\partial \phi}{\partial t}\right), \tag{1}$$

where $m = 0,\ 1$ and 2 for ISO, ICO and SO; $y = z$ for ISO and $y = r$ for ICO and SO; $\phi = M - M_e$. The above equation is subjected to the following initial and boundary conditions:

Initial condition ($t = 0$):

$$\phi(y, 0) = \phi_i = (M_i - M_e) = \text{constant}. \tag{2}$$

Boundary conditions:

At $y = 0$

$$\left(\frac{\partial}{\partial y}\phi(0, t)\right) = 0. \tag{3}$$

At $y = Y$

$$-D\left(\frac{\partial}{\partial y}\phi(Y, t)\right) = k(\phi(Y, t) - \phi_o), \tag{4}$$

where $Y = L$ or R.

Non-dimensionalizing the above equations leads to the following dimensionless parameters:

$$Bi = kY/D, \tag{5}$$

$$Fo = Dt/Y^2, \tag{6}$$

$$\Gamma = y/Y. \tag{7}$$

And the moisture content at any point of the solid object is non-dimensionalized using the following equation:

$$\Phi = (M - M_e)/(M_i - M_e). \tag{8}$$

The solution of the governing Eq. (1) and the conditions in Eqs. (2)–(4) with $\Gamma = 0$ yields dimensionless center moisture distributions for the corresponding objects in the following forms (for further details, see [5]):

$$\phi = \sum_{n=1}^{\infty} A_n B_n \quad \text{for } 0.1 < Bi < 100 \text{ and } Bi > 100. \tag{9}$$

The above solution can be simplified if the values of $(\mu_1^2 Fo) > 1.2$ are negligibly small. Thus, the infinite sum in Eq. (9) is well approximated by the first term only, i.e. (for details, see [5]):

$$\Phi \cong A_1 B_1, \tag{10}$$

where for ISO

$$A_1 = (2\sin\mu_1)/(\mu_1 + \sin\mu_1\cos\mu_1) \quad \text{or}$$
$$A_1 = G = \exp((0.2533Bi)/(1.3 + Bi)) \tag{11}$$

for ICO

$$A_1 = (2Bi)/\left((\mu_1^2 + Bi^2) + J_0(\mu_1)\right) \quad \text{or}$$
$$A_1 = G = \exp((0.5066Bi)/(1.7 + Bi)) \tag{12}$$

and for SO

$$A_1 = (2Bi\sin\mu_1)/((\mu_1) - (\sin\mu_1\cos\mu_1)) \quad \text{or}$$
$$A_1 = G = \exp((0.7599Bi)/(2.1 + Bi)) \tag{13}$$

and for all objects

$$B_1 = \exp(-\mu_1^2 Fo). \tag{14}$$

The following exponential form can express dimensionless moisture distribution:

$$\Phi = G\exp(-St), \tag{15}$$

where G represents lag factor (dimensionless) and S represents drying coefficient ($1/s$). Drying coefficient shows the drying capability of an object or product per unit time and lag factor is an indication of internal resistance of an object to the heat and/or moisture transfer during drying. These parameters are useful in evaluating and representing a drying process. The values of the dimensionless moisture content can be found using the experimental moisture content measurements from Eq. (8).

The moisture diffusivity for three regular shaped objects (infinite slab, ISO; infinite cylinder, ICO; and sphere, SP) is given by the following equation:

$$D = SY^2/\mu_1^2, \tag{16}$$

where the corresponding characteristic roots (μ_1) for ISO, ICO and SO, developed by [5] are given as

for ISO

$$\mu_1 = a\tan(0.640443Bi + 0.380397) \quad \text{for } 0 < Bi < 100 \tag{17}$$

for ICO

$$\mu_1 = ((3/4.188)\ln(6.796Bi + 1))^{1/1.4} \quad \text{for } 0 < Bi < 10, \tag{18}$$

$$\mu_1 = (\ln(1.737792Bi + 147.32))^{1/1.2} \quad \text{for } 10 < Bi < 100 \tag{19}$$

for SO

$$\mu_1 = ((1.1223)\ln(4.9Bi + 1))^{1/1.4} \quad \text{for } 0 < Bi < 10, \tag{20}$$

$$\mu_1 = ((5/3)\ln(2.199501Bi + 152.4386))^{1/1.2} \quad \text{for } 10 < Bi < 100. \tag{21}$$

Eq. (16) can easily be used to determine the moisture diffusivity values for the slab, cylindrical or spherical products. The equations determining the moisture transfer coefficients are given in the following form (for details, see [5]):

for ISO

$$k = (D/Y)[(1 - 3.94813\ln G)/(5.1325\ln G)] \tag{22}$$

for ICO

$$k = (D/Y)[(1 - 1.974\ln G)/(3.3559\ln G)] \tag{23}$$

and for SO

$$k = (D/Y)[(1 - 1.316\ln G)/(2.76369\ln G)]. \tag{24}$$

3. Results and discussion

In this section we will show how to evaluate and determine the drying process parameters and moisture transfer parameters and discuss the development of *Bi–Re* correlation. The procedure used in evaluating and determining the process parameters is clearly given in Fig. 1. It should be repeated for each case of every product accordingly.

3.1. Bi–Re correlation

The procedure mentioned above is based on experimental results. In order to find the drying process parameters, we need to measure the center moisture content values or distribution. However, in practice design engineers and workers prefer to use tools, models, correlations, charts, etc., for design and optimization of the process, which not only saves their valuable time but also helps in evaluating these parameters in a simple and accurate way without measuring the moisture values.

Because of the thermophysical properties and velocity of the drying fluid, it is known to have a relation between Biot number and Reynolds number (UY/v).

Using the experimental drying data taken from various literature sources for more than 20 different products, we have obtained Fig. 2 as the Biot number vs Reynolds number chart with the following *Bi–Re* correlation in the form of $Bi = aRe^{-b}$ for food products subject to air drying with a correlation coefficient of 0.72 as follows:

$$Bi = 22.55Re^{-0.59}. \tag{25}$$

3.2. Illustrative example

The main goal of this illustrative example is to show how the present methodology is used to determine the moisture process parameters, e.g., lag factors and drying coefficients and the moisture transfer parameters, e.g., moisture diffusivities and moisture transfer coefficients for three regular shaped products, e.g., infinite slab, infinite cylinder and sphere, using the experimental moisture content data. In order to validate the applicability of the present correlation in solids drying processes, we employ the experimental average moisture content distributions of three different products such as slab shaped prune as ISO, cylindrically shaped potato as ICO and spherically shaped potato as SO. The experimental drying conditions and their characteristic dimensions such as half thickness for ISO and radius for ICO and SO are listed in Table 1.

The following procedure is employed to determine the moisture transfer parameters and moisture content distributions of the products:

1. The kinematic viscosity values for the experimental conditions were taken from [8].
2. The Reynolds numbers were calculated for ISO, ICO and SO, respectively.

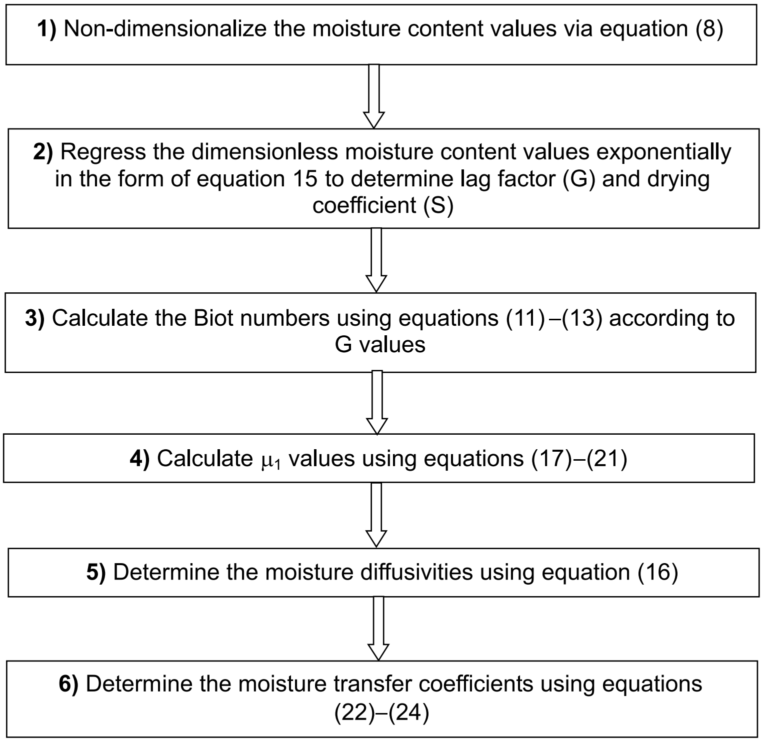

Fig. 1. Procedure used in evaluating and determining the process parameters.

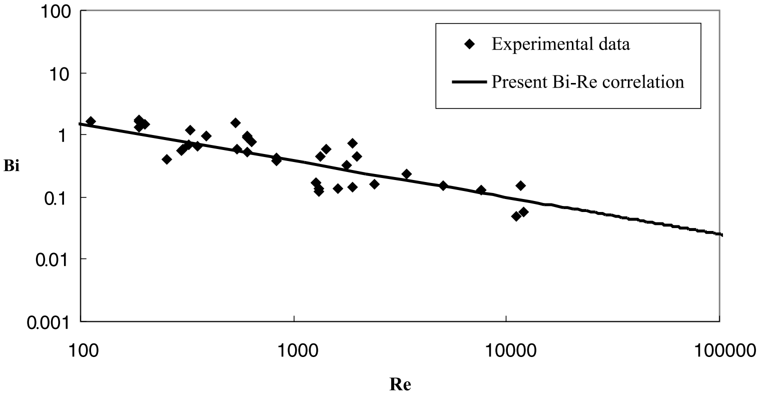

Fig. 2. Bi–Re diagram for food products subject to air drying.

Table 1
Experimental conditions and characteristic dimensions of the products

Shape	ISO	ICO	SO
Product	Prune	Potato	Potato
Air temperature (°C)	60	60	40
Air velocity (m/s)	3.5	1.0	1.0
Characteristic dimension (m)	0.0025	0.0135	0.009
Source	[9]	[11]	[10]

3. Using the correlations in Eqs. (11)–(13), the Biot numbers were estimated for these three products.
4. The characteristic roots were calculated using the relations in Eqs. (17)–(21) for different products.
5. The moisture diffusivities were then calculated using the model in Eq. (16).
6. The moisture transfer coefficients were calculated using Eqs. (22)–(24).
7. Finally, dimensionless moisture distributions were obtained by evaluating A_1's and B_1's in Eqs. (11)–(14) and substituting in Eq. (10).

The values of the drying coefficient (S), lag factor (G), Bi, root of the characteristic equation (μ), moisture diffusivity (D) and moisture transfer coefficient (k) for the slab, cylindrical and spherical products were obtained through the above listed methodology and these values are tabulated in Table 2. It is important to point out that the moisture content data of these three products were not utilized in the development of the present correlation in order to avoid any bias for the present work.

Here, we now verify the applicability of the Bi–Re correlation for the three respective objects. In this regard, using the obtained data in Table 2 we calculate the dimensionless moisture content distributions for these slab, cylindrical and spherical products subject to drying at different conditions and compare these calculated dimensionless moisture content profiles with the experimental dimensionless average moisture content values. Figs. 3–5 show the moisture profiles for ISO, ICO and SO objects subjected to drying in air. The obtained moisture profiles are in good agreement with the experimental data taken from the literature. From the figures, it is clear that the dimensionless moisture profiles obtained from the present correlation agree well with the experimental moisture content data. The average error between the results of the present correlation and experimental data is ±7.1% for ISO, ±4.0% for ICO and ±9.8% for SO. In the figures, we also plot the values calculated using a recent mass transfer correlation, i.e., Sherwood–Reynolds correlation, developed by El-Naas et al. [7], particularly for spouted bed dryers, as follows:

$$Sh = 0.000258Re^{1.66}, \tag{26}$$

where $Sh = kY/D$. Note that in moisture transfer applications we can take the Biot number given in Eq. (5) same as the Sherwood number in Eq. (26).

Table 2
Thermal parameters of the products obtained through the present methodology

Thermal parameters	Product		
	ISO	ICO	SO
S (1/s)	0.0003	7×10^{-5}	0.0009
G (dimensionless)	1.0037	1.032	1.0074
Re (dimensionless)	910.9933	1405.5325	1046.6455
Bi (dimensionless)	0.4054	0.3139	0.3736
μ_1	0.1674	0.4358	0.2781
D (m^2/s)	6.6905×10^{-8}	6.7172×10^{-8}	9.4198×10^{-7}
k (m/s)	1.0849×10^{-5}	1.5618×10^{-6}	3.9102×10^{-5}

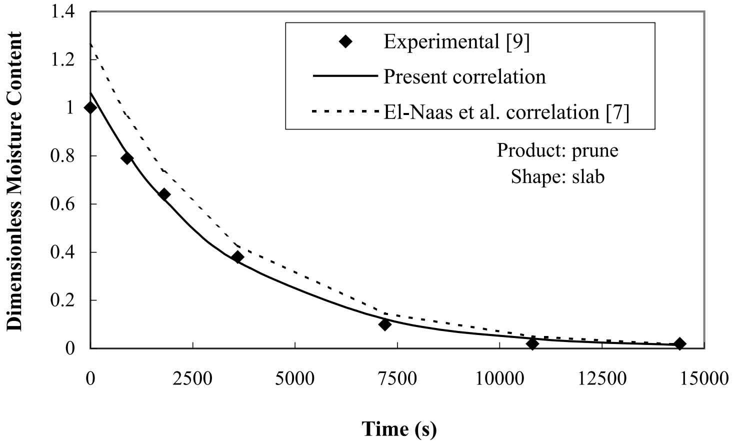

Fig. 3. Experimental and calculated dimensionless moisture content distributions of ISO.

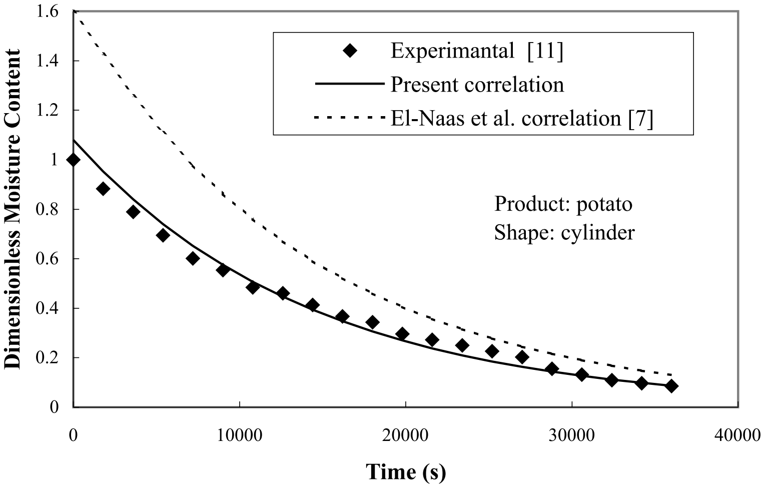

Fig. 4. Experimental and calculated dimensionless moisture content distributions of ICO.

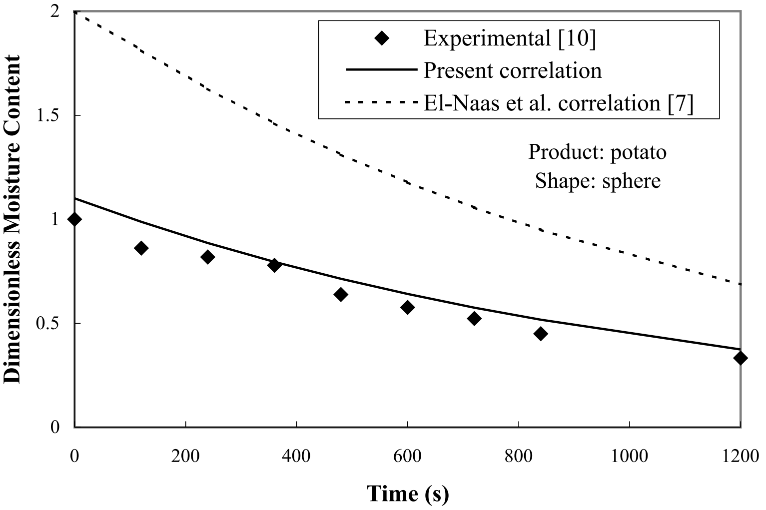

Fig. 5. Experimental and calculated dimensionless moisture content distributions of SO.

As clearly shown in the figures, the dimensionless moisture profiles obtained from the present correlation are much closer to the experimental data than the results calculated from the correlation of El-Naas et al. [7].

In summary, the present correlation (Eq. 25) can be proposed as a significant expression for practical applications since in practice it is sometimes extremely difficult to conduct experimental measurements for design purposes. Therefore, people need simple correlations or tools that they can be used easily without getting into complicated analysis. It is believed that the present correlation is in this regard considerably beneficial to the drying people.

4. Conclusion

The present paper discussed the development of a new drying correlation for products subjected to drying. The developed correlation used to determine the moisture transfer parameters, i.e., moisture diffusivity and moisture transfer coefficients was presented with illustrative examples. Moreover, dimensionless center moisture distribution was obtained for ISO, ICO and SO, respectively. The results obtained were compared with the experimental data and the correlation available in the literature and found to be in good agreement with the experiments compared to the other correlation. Thus, the present correlation can be used with reasonable accuracy and confidence for such drying applications.

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References

[1] C.-C. Jia, D.-W. Sun, C.-W. Cao, Mathematical simulation of temperature and moisture fields within a grain kernel during drying, Drying Technol. 18 (2000) 1305–1325.

[2] M.A. Ruiz-Cabrera, M.A. Salgado-Cervantes, K.N. Waliszewski-Kubiak, M.A. Garcia-Alvarado, The effect of path diffusion on the effective diffusivity in carrot slabs, Drying Technol. 15 (1) (1997) 169–181.

[3] T. Akiyama, H. Liu, K. Hayakawa, Hygrostress multicrack formation and propagation in cylindrical viscoelastic food undergoing heat and moisture transfer processes, Int. J. Heat Mass Transfer 40 (7) (1997) 1601–1609.

[4] S.K. Chou, M.N.A. Hawlader, K.J. Chua, Identification of the receding evaporation front in convective food drying, Drying Technol. 15 (1997) 1353–1367.

[5] I. Dincer, S. Dost, A modeling study for moisture diffusivities and moisture transfer coefficients in drying of solid objects, Int. J. Energy Res. 20 (1996) 531–539.

[6] I. Dincer, A.Z. Sahin, B.S. Yilbas, A.A. Al-Farayedhi, M.M. Hussain, Exergy and energy analysis of food drying systems, Progress Report 2, KFUPM Project #ME/ENERGY/203, 2000.

[7] M.H. El-Naas, S. Rognon, R. Legros, R.C. Mayer, Hydrodynamics and mass transfer in a spouted bed dryer, Drying Technol. 18 (2000) 323–340.

[8] I. Dincer, Heat Transfer in Food Cooling Applications, Taylor & Francis, Washington, DC, 1997.

[9] E. Tsami, M. Katsioti, Drying kinetics for some fruits: predicting of porosity and color during drying, Drying Technol. 18 (2000) 559–1581.

[10] C.P. McLaughlin, T.R.A. Magee, The effects of air temperature, sphere diameter and puffing with CO_2 on the drying of potato spheres, Drying Technol. 17 (1999) 119–136.

[11] W.A.M. McMinn, T.R.A. Magee, Air drying of potato cylinders, Drying Technol. 14 (1996) 2025–2040.